ENAMA - Encontro Nacional de Análise Matemática e Aplicações
IME-USP - Instituto de Matemática e Estatística da Universidade de São Paulo
XV ENAMA - Novembro 2022 *107–108*

ABOUT THE CAUCHY PROBLEM FOR THE SYSTEM WITH THREE SCHRODINGER EQUATIONS WITH QUADRATIC NONLINEARITY

ISNALDO ISAAC BARBOSA[1] & JEFFERSON DA ROCHA SILVA[2]

[1]Instituto de Matemática, UFAL, AL, Brasil,isnaldo@pos.mat.ufal.br,

[2]Instituto de Física, UFAL, AL, Brasil, rochasilvajrs@gmail.com

Abstract

We study the Good Local Placement of the Cauchy Problem described by the coupling of three Schrodinger equations in the continuous case in dimension 1 presented in (1).

This work is inspired by the results obtained in [1] for a coupled system of two Schrodinger equations with quadratic nonlinearity. This work aims to study the Cauchy problem for a coupled system of equations type Schrodinger on the real straight.

To establish the main result of this work, we obtain inequalities in Bougain spaces.

1 Introduction

This work is dedicated to studying the Cauchy problem for a system of equations that arises in nonlinear optics problems. More precisely, let us study the following mathematical model:

$$\begin{cases} iw_t + w_{xx} - w + \bar{w}v + \bar{v}u = 0 \\ 2iv_t + v_{xx} - \beta v + \frac{1}{2}w^2 + \bar{w}u = 0 \\ 3iu_t + u_{xx} - \beta_1 u + \chi vw = 0 \\ w(x,0) = w_0(x) \in H^r(\mathbb{R}),\ v(x,0) = v_0(x) \in H^s(\mathbb{R}) \text{ e } u(x,0) = u_0(x) \in H^k(\mathbb{R}), \end{cases} \tag{1}$$

where w, v, and u are functions that take on complex values and represent the fundamental harmonic, second, and third harmonic, respectively, and β, β_1, and χ are real numbers that represent the physical parameters of the system. For more physical information on this model, we recommend [2].

In [3], results of global local good placement are established for the system (1) in the cases $r = s = k = 0$ or 1, that is, in L^2 and H^1. In addition, stability results are obtained for traveling wave solutions. Finally, our result extends the region of local well-posed obtained in [3] concerning the Sobolev indices.

2 Main Results

Theorem 2.1. *Given $(w_0, v_0, u_0) \in H^r(\mathbb{R}) \times H^s(\mathbb{R}) \times H^k(\mathbb{R})$ with $(r, s, k) \in \mathcal{R} \subset \mathbb{R}^3$. The Cauchy problem (1) is locally well-posed in $H^r(\mathbb{R}) \times H^s(\mathbb{R}) \times H^k(\mathbb{R})$ in the following way: for each $\rho > 0$, exist $T = T(\rho) > 0$ and $b > 1/2$ such that for all initial data with $\|w_0\|_{H^r} + \|v_0\|_{H^s} + \|u_0\|_{H^k} < \rho$, there is only one solution (w, v, u) for (1) satisfying the following conditions:*

$$\psi_T(t)w \in X^{r,b}, \quad \psi_T(t)v \in X_2^{s,b} \quad \textit{and} \quad \psi_T(t)w \in X_3^{k,b},$$
$$w \in C([0,T]; H^r), \quad v \in C([0,T]; H^s) \quad \textit{and} \quad u \in C\left([0,T]; H^k\right).$$

In addition, the data-solution application is locally Lipschitzian.

Proof The proof of the Theorem above simulates the proof presented in section IV of [1].

The key results for establishing this Theorem and delimiting the $\mathcal{R}$ region of the Main Theorem are:

(i) $\|\overline{w}\cdot v\|_{X^{r,-d}} \leq C\|w\|_{X^{r,b}}\cdot\|v\|_{X_2^{s,b}}$, with (r,s,k) satisfying a condition $\mathcal{R}_1$;

(ii) $\|\overline{v}\cdot u\|_{X^{r,-d}} \leq C\|v\|_{X_2^{s,b}}\cdot\|u\|_{X_3^{k,b}}$ with (r,s,k) satisfying a condition $\mathcal{R}_2$;

(iii) $\|w\cdot\tilde{w}\|_{X_2^{s,-d}} \leq C\|w\|_{X^{r,b}}\cdot\|w\|_{X^{r,b}}$ with (r,s,k) satisfying a condition $\mathcal{R}_3$;

(iv) $\|\overline{w}\cdot u\|_{X_2^{s,-d}} \leq C\|w\|_{X^{r,b}}\cdot\|u\|_{X_3^{k,b}}$ with (r,s,k) satisfying a condition $\mathcal{R}_4$ e

(v) $\|w\cdot v\|_{X_3^{k,-d}} \leq C\|w\|_{X^{r,b}}\cdot\|v\|_{X_2^{s,b}}$ with (r,s,k) satisfying a condition $\mathcal{R}_5$.

The region presented in the theorem is $\mathcal{R}=\mathcal{R}_1\cap\mathcal{R}_2\cap\mathcal{R}_3\cap\mathcal{R}_4\cap\mathcal{R}_5$.

The inequalities (i) and (iii) are obtained in [1].

The Bougain space, $X_j^{a,b}$ considered in this work is the completion of $\mathcal{S}(\mathbb{R}^2)$ with respect to the norm:

$$\|f\|_{X_j^{a,b}} = \left\|\langle\xi\rangle^a\langle j\tau+\phi(\xi)\rangle^b\widehat{f}(\tau,\xi)\right\|_{L^2_\tau L^2_\xi},$$

we consider $X_1^{a,b}=X^{a,b}$.

The other inequalities are under review, therefore, this work is under writing for submission.

References

[1] Barbosa, I.I. The Cauchy problem for nonlinear quadratic interactions of the Schrödinger type in one dimensional space, *Journal of Mathematical Physics*, **59**, 7,2018

[2] Kivshar, Y. S. et al - Multi-component optical solitary waves, *Physica A: Statistical Mechanics and its Applications* **228**,2000

[3] Pastor, A. - On tree-wave interaction Schrodinger Systems with quadratic nonlinearities: Global Well-Posedness and Stamding Waves, *Communications on Pure and Applied Analysis* **18**, 5,2019